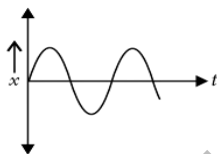


## CHAPTER: Oscillations

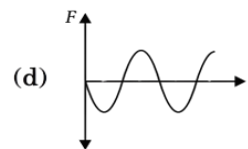
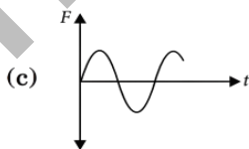
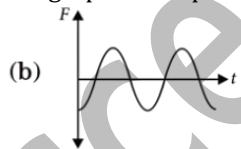
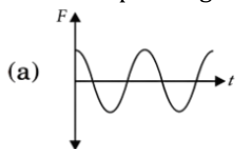
### OBJECTIVE TYPE QUESTIONS

#### Multiple Choice Questions (MCQs)

- Which of the following relationships between the acceleration  $a$  and the displacement  $x$  of a particle involve simple harmonic motion?  
 (a)  $a = 0.7x$                       (b)  $a = -200x^2$                       (c)  $a = -10x$                       (d)  $a = 100x^3$
- Which of the following examples represent periodic but not simple harmonic motion?  
 (a) The rotation of earth about its axis.  
 (b) Motion of an oscillating mercury column in a U-tube.  
 (c) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.  
 (d) All of the above
- The function  $\sin \omega t - \cos \omega t$  represents  
 (a) a simple harmonic motion with a period  $\frac{\pi}{\omega}$ .  
 (b) a simple harmonic motion with a period  $\frac{2\pi}{\omega}$ .  
 (c) a periodic, but not simple harmonic motion with a period  $\frac{\pi}{\omega}$ .  
 (d) a periodic, but not simple harmonic motion with a period  $\frac{2\pi}{\omega}$ .
- Displacement-time graph of a particle executing S.H.M. is shown below.



The corresponding force-time graph of the particle can be



- The maximum speed of a particle executing S.H.M. is 10 m/s and maximum acceleration is  $31.4 \text{ m/s}^2$ . Its periodic time is  
 (a) 2 s                      (b) 4 s                      (c) 6 s                      (d) 1 s

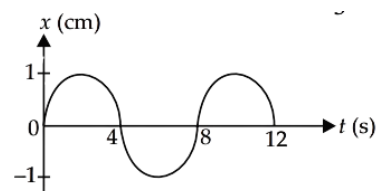
6 The  $x - t$  graph of a particle undergoing simple harmonic motion is shown in figure. The acceleration of the particle at  $t = \frac{4}{3} \text{ s}$  is

(a)  $\frac{\sqrt{3}}{32} \pi^2 \text{ cm s}^{-2}$

(b)  $-\frac{\pi^2}{32} \text{ cm s}^{-2}$

(c)  $\frac{\pi^2}{32} \text{ cm s}^{-2}$

(d)  $-\frac{\sqrt{3}}{32} \pi^2 \text{ cm s}^{-2}$



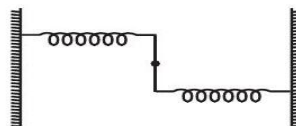
- A particle executes SHM of type  $x = A \sin \omega t$ . It takes time  $t_1$  from  $x = 0$  to  $x = \frac{A}{2}$  and  $t_2$  from  $x = \frac{A}{2}$  to  $x = A$ . The ratio  $t_1 : t_2$  will be  
 (a) 1:1                      (b) 1:2                      (c) 1:3                      (d) 2:1

8 A particle is executing S.H.M. according to the equation  $x = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$  in SI units. The displacement and acceleration of the particle at  $t = 1.5$  s is

- (a)  $-3.0$  m,  $100$  m s<sup>-2</sup>                      (b)  $+2.54$  m,  $200$  m s<sup>-2</sup>  
 (c)  $-3.54$  m,  $140$  m s<sup>-2</sup>                      (d)  $+3.55$  m,  $120$  m s<sup>-2</sup>

9 A particle executes a linear S.H.M. with an amplitude of 4 cm. At the mean position the velocity of the particle is 10 cm/s. What is the displacement of the particle when its speed becomes 5 cm/s ?

- (a)  $2(\sqrt{3})$  cm  
 (b)  $2(\sqrt{5})$  cm  
 (c)  $\sqrt{5}$  cm  
 (d)  $\sqrt{3}$  cm

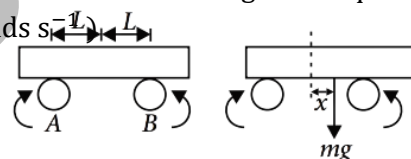


10. A uniform rod of length  $l$  and mass  $M$  is pivoted at the centre. Its two ends are attached to two springs of equal spring constant  $k$ . The springs are fixed to rigid support as shown in figure and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle  $\theta$  in one direction and released. The frequency of oscillation is

- (a)  $\frac{1}{2\pi}\sqrt{\frac{2k}{6M}}$                       (b)  $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$                       (c)  $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$                       (d)  $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

11 A uniform bar with mass  $m$  lies symmetrically across two rapidly rotating fixed rollers,  $A$  and  $B$  with distance  $L = 2.0$  cm between the bar's centre of mass and each roller. The rollers, whose directions of rotation are shown in figures slip against the bar with coefficient of kinetic friction  $\mu_k = 0.40$ . Suppose the bar is displaced horizontally by a distance  $x$  as shown in figure and then released. The angular frequency  $\omega$  of the resulting horizontal simple harmonic motion of the bar is (in rads s<sup>-1</sup>)

- (a) 14  
 (b) 15  
 (c) 16  
 (d) 17



12 A particle performs simple harmonic motion with a period of 2 seconds. The time taken by it to cover a displacement equal to half of its amplitude from the mean position is

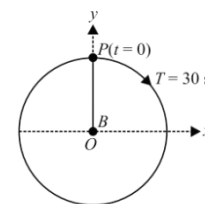
- (a)  $1/2$  s                      (b)  $1/3$  s                      (c)  $1/4$  s                      (d)  $1/6$  s

13 A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of  $100$  N m<sup>-1</sup>. The block is pulled to a distance  $x = 10$  cm from its equilibrium position at  $x = 0$  on a frictionless surface from rest at  $t = 0$ . The kinetic energy and potential energy of the block when it is 5 cm away from the mean position is

- (a) 0.375 J, 0.125 J    (b) 0.125 J, 0.375 J    (c) 0.125 J, 0.125 J    (d) 0.375 J, 0.375 J

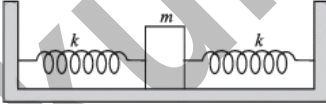
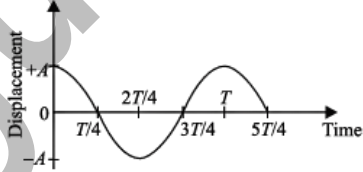
14 Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the  $x$ -projection of the radius vector of the rotating particle  $P$  is

- (a)  $x(t) = A\sin\left(\frac{2\pi}{30}t\right)$                       (b)  $x(t) = A\cos\left(\frac{\pi}{15}t\right)$   
 (c)  $x(t) = A\sin\left(\frac{\pi}{15}t + \frac{\pi}{2}\right)$                       (d)  $x(t) = A\cos\left(\frac{\pi}{15}t + \frac{\pi}{2}\right)$

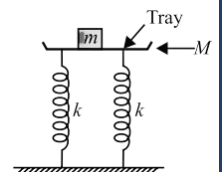
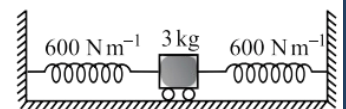


15 The displacement of a particle varies with time according to the relation:  $y = a\sin \omega t + b\cos \omega t$ .

- (a) The motion is oscillatory but not SHM.  
 (b) The motion is SHM with amplitude  $a + b$ .

- (c) The motion is SHM with amplitude  $a^2 + b^2$ .  
 (d) The motion is SHM with amplitude  $\sqrt{a^2 + b^2}$
- 16 Four pendulums  $A, B, C$  and  $D$  are suspended from the same elastic support as shown in figure.  $A$  and  $C$  are of the same length, while  $B$  is smaller than  $A$  and  $D$  is larger than  $A$ . If  $A$  is given a transverse displacement then
- $D$  will vibrate with maximum amplitude
  - $C$  will vibrate with maximum amplitude
  - $B$  will vibrate with maximum amplitude
  - all the four will oscillate with equal amplitude.
- 17 Two identical springs of spring constant  $k$  are attached to a block of mass  $m$  and to fixed supports as shown in the figure. The time period of oscillation is
- 
- $2\pi\sqrt{\frac{m}{k}}$
  - $2\pi\sqrt{\frac{m}{2k}}$
  - $2\pi\sqrt{\frac{2m}{k}}$
  - $\pi\sqrt{\frac{m}{2k}}$
18. The displacement-time graph for a particle executing SHM is as shown in figure. Which of the following statements is correct?
- 
- The velocity of the particle is maximum at  $t = \frac{3}{4}T$
  - The velocity of the particle is maximum at  $t = \frac{T}{2}$
  - The acceleration of the particle is maximum at  $t = \frac{T}{4}$
  - The acceleration of the particle is maximum at  $t = \frac{3}{4}T$
- 19 Which of the following examples represent periodic motion as well as simple harmonic motion?
- A swimmer completing one (return) trip from one bank of a river to the other and back.
  - A freely suspended bar magnet displaced from its  $N - S$  direction and released.
  - A hydrogen molecule rotating about its centre of mass.
  - An arrow released from a bow.
- 20 A particle executing SHM has a maximum speed of  $30 \text{ cm s}^{-1}$  and a maximum acceleration of  $60 \text{ cm s}^{-2}$ . The time period of oscillation is
- $\pi \text{ s}$
  - $\frac{\pi}{2} \text{ s}$
  - $\frac{\pi}{4} \text{ s}$
  - $2\pi \text{ s}$
- 21 A particle executing SHM is described by the displacement function  $x(t) = A \cos(\omega t + \phi)$ , If the initial ( $t = 0$ ) position of the particle is  $1 \text{ cm}$ , its initial velocity is  $\pi \text{ cm s}^{-1}$  and its angular frequency is  $\pi \text{ s}^{-1}$ , then the amplitude of its motion is
- $\pi \text{ cm}$
  - $2 \text{ cm}$
  - $\sqrt{2} \text{ cm}$
  - $1 \text{ cm}$
- 22 When the displacement of a particle executing SHM is one-fourth of its amplitude, what fraction of the total energy is the kinetic energy?
- $\frac{16}{15}$
  - $\frac{15}{16}$
  - $\frac{3}{4}$
  - $\frac{4}{3}$
- 23 If the frequency of human heart beat is  $1.25 \text{ Hz}$ , the number of heart beats in 1 minute is
- 80
  - 65
  - 90
  - 75

- 24 A point particle oscillates along the  $x$ -axis according to the law  $x = x_0 \cos \left( \omega t - \frac{\pi}{4} \right)$ . If the acceleration of the particle is written as  $a = A \cos (\omega t + \delta)$ , then  
 (a)  $A = x_0 \omega^2, \delta = 3\pi/4$  (b)  $A = x_0, \delta = -\pi/4$   
 (c)  $A = x_0 \omega^2, \delta = \pi/4$  (d)  $A = x_0 \omega^2, \delta = -\pi/4$
- 25 The length of a second's pendulum on the surface of earth is 1 m. Its length on the surface of the moon is  
 (a)  $\frac{1}{6}$  m (b) 1 m (c)  $\frac{1}{36}$  m (d) 36 m
- 26 A block of mass 200 g executing SHM under the influence of a spring of spring constant  $k = 90 \text{ N m}^{-1}$  and a damping constant  $b = 40 \text{ g s}^{-1}$ . The time elapsed for its amplitude to drop to half of its initial value is (Given  $\ln (1/2) = -0.693$ )  
 (a) 7 s (b) 9 s (c) 4 s (d) 11 s
- 27 A particle executes simple harmonic motion between  $x = -A$  and  $x = +A$ . The time taken for it to go from 0 to  $A/2$  is  $T_1$  and to go from  $A/2$  to  $A$  is  $T_2$ . Then  
 (a)  $T_1 < T_2$  (b)  $T_1 > T_2$  (c)  $T_1 = T_2$  (d)  $T_1 = 2T_2$
- 28 Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?  
 (a)  $\frac{1}{6}$  s (b)  $\frac{1}{4}$  s (c)  $\frac{1}{3}$  s (d)  $\frac{1}{12}$  s
- 29 Consider a pair of identical pendulums, which oscillate with equal amplitude independently such that when one pendulum is at its extreme position making an angle of  $2^\circ$  to the right with the vertical, the other pendulum makes an angle of  $1^\circ$  to the left of the vertical. The phase difference between the pendulums is  
 (a)  $\frac{\pi}{2}$  (b)  $\frac{2}{3}\pi$  (c)  $\frac{3}{2}\pi$  (d)  $\pi$
- 30 A simple pendulum executing SHM with a period of 6 s between two extreme positions  $B$  and  $C$  about a point  $O$ . If the length of the arc  $BC$  is 10 cm, how long will the pendulum take to move from position  $C$  to a position  $D$  towards  $O$  exactly midway between  $C$  and  $O$  ?  
 (a) 0.5 s (b) 1 s (c) 1.5 s (d) 3 s
- 31 A trolley of mass 3 kg, as shown in figure, is connected to two identical springs, each of spring constant  $600 \text{ N m}^{-1}$ . If the trolley is displaced from its equilibrium position by 5 cm and released, the maximum speed of the trolley is  
 (a)  $0.5 \text{ m s}^{-1}$  (b)  $1 \text{ m s}^{-1}$  (c)  $2 \text{ m s}^{-1}$  (d)  $3 \text{ m s}^{-1}$
- 32 A tray of mass  $M = 10 \text{ kg}$  is supported on two identical springs, each of spring constant  $k$ , as shown in figure. When the tray is depressed a little and released, it executes simple harmonic motion of period 1.5 s. When a block of mass  $m$  is placed on the tray, the period of oscillation becomes 3 s. The value of  $m$  is  
 (a) 10 kg (b) 20 kg (c) 30 kg (d) 40 kg
- 33 The amplitude of a damped oscillator becomes  $\left(\frac{1}{3}\right)^{\text{rd}}$  in 2 seconds. If its amplitude after 6 seconds is  $\frac{1}{n}$  times the original amplitude, the value of  $n$  is  
 (a)  $3^2$  (b)  $\sqrt[3]{3}$  (c)  $2^3$  (d)  $3^3$



- 34 A simple pendulum has time period  $T_1$ . The point of suspension is now moved upward according to the relation  $y = Kt^2$ . ( $K = 1 \text{ m s}^{-2}$ ) where  $y$  is the vertical displacement. The time period now becomes  $T_2$ . The ratio of  $\frac{T_1^2}{T_2^2}$  is (Take  $g = 10 \text{ m s}^{-2}$ )
- (a)  $\frac{6}{5}$                       (b)  $\frac{5}{6}$                       (c) 1                      (d)  $\frac{4}{5}$
- 35 An air chamber of volume  $V$  has a neck of cross-sectional area  $a$  into which a light ball of mass  $m$  just fits and can move up and down without friction. The diameter of the ball is equal to that of the neck of the chamber. The ball is pressed down a little and released. If the bulk modulus of air is  $B$ , the time period of the oscillation of the ball is
- (a)  $T = 2\pi \sqrt{\frac{Ba^2}{mV}}$                       (b)  $T = 2\pi \sqrt{\frac{BV}{ma^2}}$                       (c)  $T = 2\pi \sqrt{\frac{mB}{Va^2}}$                       (d)  $T = 2\pi \sqrt{\frac{mV}{Ba^2}}$
- 36 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A block of mass  $m$  is suspended from this balance, when displaced and released, it oscillates with a period 0.5 s. The value of  $m$  is (Take  $g = 10 \text{ m s}^{-2}$ )
- (a) 8 kg                      (b) 12 kg                      (c) 16 kg                      (d) 20 kg
- 37 A U-tube of uniform cross-section holds 1 kg of pure mercury and 0.2 kg of water in equilibrium. The diameter of cross-section is 1.2 cm. Relative density of mercury is 13.6. If the system in equilibrium is slightly disturbed, the period of oscillation of the liquid column in the tube will be (take  $g = 10 \text{ m s}^{-2}$ )
- (a) 0.1 s                      (b) 1.5 s                      (c) 0.50 s                      (d) 0.2 s

### 3. Case Based MCQs

*Case I : Read the passage given below and answer the following questions from 38 to 42 .*

#### 4. Simple Harmonic Motion

Simple harmonic motion is the simplest form of oscillation. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (S.H.M).

A body is undergoing simple harmonic motion if it has an acceleration which is directed towards a fixed point, and proportional to the displacement of the body from that point.

Acceleration  $a \propto -x \Rightarrow a = -kx$  or  $\frac{d^2x}{dt^2} = -kx$ , where  $x$  = displacement at any instant  $t$ .

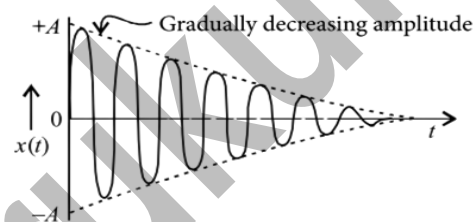
- 38 Which of the following is not a characteristic of simple harmonic motion?
- (a) The motion is periodic.  
 (b) The motion is along a straight line about the mean position.  
 (c) The oscillations are responsible for the energy conversion.  
 (d) The acceleration of the particle is directed towards the extreme position.
- 39 The equation of motion of a simple harmonic motion is
- (a)  $\frac{d^2x}{dt^2} = -\omega^2 x$                       (b)  $\frac{d^2x}{dt^2} = -\omega^2 t$                       (c)  $\frac{d^2x}{dt^2} = -\omega x$                       (d)  $\frac{d^2x}{dt^2} = -\omega t$
- 40 Which of the following expressions does not represent simple harmonic motion?
- (a)  $x = A \cos \omega t + B \sin \omega t$                       (b)  $x = A \cos (\omega t + \alpha)$   
 (c)  $x = B \sin (\omega t + \beta)$                       (d)  $x = A \sin \omega t \cos^2 \omega t$

- 41 The time period of simple harmonic motion depends upon  
 (a) amplitude (b) energy (c) phase constant (d) mass
- 42 Which of the following motions is not simple harmonic?  
 (a) Vertical oscillations of a spring (b) Motion of a simple pendulum  
 (c) Motion of planet around the Sun (d) Oscillation of liquid in a U-tube

**Case II: Read the passage given below and answer the following questions from 43 to 46 .**

### 5. Damped Simple Harmonic Motion

The oscillations in presence of dissipative force where the amplitude decreases gradually with the passage of time are called damped oscillations. A part of the energy of the oscillating system is lost in the form of heat, in overcoming these resistive forces, as a result, the amplitude of such oscillations decreases exponentially with time, as shown in figure. Eventually, these oscillations die out. In these oscillations, the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, etc. Due to decrease in amplitude, the energy of the oscillator also goes on decreasing exponentially.



The force producing a resistance to the oscillation is called damping force.

- 43 A particle oscillating under a force  $\vec{F} = -k\vec{x} - b\vec{v}$  is a ( $k$  and  $b$  are constants)  
 (a) simple harmonic oscillator (b) linear oscillator  
 (c) damped oscillator (d) forced oscillator
- 44 Which of the following displacement-time graphs represent damped harmonic oscillation?
- (a) (b) (c) (d)

- 45 In case of a force vibration, the resonance wave becomes very sharp when the  
 (a) applied periodic force is small (b) quality factor is small  
 (c) damping force is small (d) restoring force is small

- 46 The S.I. unit of damping constant is  
 (a) kgs (b)  $\text{kg}^2 \text{ s}$  (c)  $\text{kgm/s}$  (d)  $\text{kg/s}$

### 6. Assertion & Reasoning Based MCQs

**For question numbers 47-56, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.**

- (a) Both A and R are true and R is the correct explanation of A  
 (b) Both A and R are true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false and R is also false
- 47 **Assertion (A)** : Resonance is a special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency and the amplitude of forced vibration, is maximum.

**Reason (R) :** The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

48 **Assertion (A) :** Simple harmonic motion is not a uniformly accelerated motion.

**Reason (R) :** Velocity is non uniform in SHM.

49 **Assertion (A) :** The bob of a simple pendulum is a ball full of water. If a fine hole is made at the bottom of the ball, then the time period will no more remain constant.

**Reason (R) :** The time period of a simple pendulum does not depend upon mass.

50 **Assertion (A) :** When a simple pendulum is made to oscillate on the surface of moon, its time period increases.

**Reason (R) :** Moon is much smaller as compared to earth.

51 **Assertion (A) :** If a man with a wrist watch on his hand falls from the top of a tower, its watch gives correct time during the free fall.

**Reason (R) :** The working of the wrist watch depends on spring action and it has nothing to do with gravity.

52 **Assertion (A) :** The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%.

**Reason (R) :** Time period is directly proportional to length of pendulum.

53 **Assertion (A) :** The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is  $0.612 \text{ s}^{-1}$ .

**Reason (R) :** The frequency of a second pendulum does not depend upon acceleration due to gravity.

54 **Assertion (A) :** Damped oscillation indicates loss of energy.

**Reason (R) :** The loss in damped oscillation may be due to friction, air resistance etc.

55 **Assertion (A) :** The value of acceleration due to gravity is low at the mountain top than at the plane.

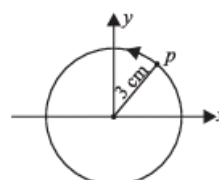
**Reason (R) :** If a pendulum clock is taken to mountain top, it will gain time.

56 **Assertion (A) :** In real situation the amplitude of forced oscillation can never be infinite.

**Reason (R) :** The energy of oscillator is continuously dissipated.

### Very Short Answer Type Questions (VSA)

- 1 What are the basic properties required by a system to oscillate?
- 2 What is the : (a) distance moved (b) displacement of a particle executing SHM in one vibration?
- 3  $x(t) = A \cos(\omega t + \phi)$  is the equation of simple harmonic motion. What do we call  $\phi$  in this equation?
- 4 The figure shows circular motion of a reference particle to represent simple harmonic motion. What is the amplitude of simple harmonic motion?



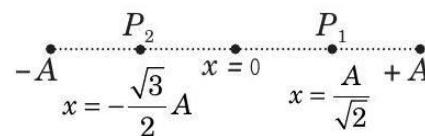
- 5 A simple harmonic motion is described by  $a = -16x$  where  $a$  is acceleration and  $x$  is displacement in meter. What is the time-period?
- 6 Glass window panes are sometimes broken by a explosion several miles away. Explain why?
- 7 On what factors does the energy of a simple harmonically vibrating particle depends?
- 8 How is the frequency of oscillation related with the frequency of change in the of K.E and P.E of the body in S.H.M.?
- 9 What will be the time period of a second pendulum inside an artificial satellite?
- 10 Two simple pendulums of unequal length meet each other at mean position while oscillating. What is their phase difference?

### Short Answer Type Questions (SA-I)

- 11 How is the time period of the pendulum effected when pendulum is taken to hills or in mines?
- 12 Two exactly identical pendulums are executing (approximate) SHMs with amplitudes  $a$  and  $na$  respectively. Calculate the ratio of their energies of oscillation.
- 13 A block weighing 40 N is suspended from a spring that has a force constant of  $200\text{Nm}^{-1}$ . The system is undamped and is subjected to a harmonic driving force of frequency 10 Hz, resulting a forced-motion amplitude of 2 cm. What is the maximum value of driving force?
- 14 A 2 kg particle undergoes SHM according to  $x = 1.5\sin\left(\frac{\pi}{4}t + \frac{\pi}{6}\right)$ , when  $x$  is in metre and  $t$  in second. What is the total mechanical energy of the particle?
- 15 Show that the acceleration of a particle in SHM is proportional to its displacement from the mean position.
- 16 Three springs of spring factor  $k, 2k, k$ , respectively are connected in parallel to a mass  $m$ . If the mass = 0.08 kg and  $k = 2\text{ N/m}$ , then find the new time period?
- 17 Three springs of constants  $k_1, k_2$  and  $k_3$  are connected end to end and are used to hang a mass  $M$  from the roof. Determine the elastic potential energy stored in the array.
- 18 The maximum velocity of a body undergoing simple harmonic motion is  $0.04\text{ m s}^{-1}$  and its acceleration at 0.02 m from the mean position is  $0.06\text{ m s}^{-2}$ . Determine its amplitude and time period.
- 19 The formula for time period  $T$  for a loaded spring,  $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$ . Does the time period depend on the length of the spring?
- 20 Figure shows the acceleration-displacement graph of a particle in SHM. Find the time period (in second).

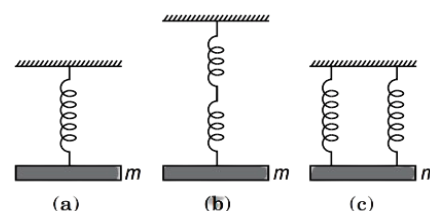
**Short Answer Type Questions (SA-II)**

- 21 Two particles  $P_1$  and  $P_2$  are executing SHM along the same straight line, whose equations are given  $x_1 = A \sin(\omega t + \delta_1)$  and  $x_2 = A \sin(\omega t + \delta_2)$ . An observer on the ground, at  $t = 0$ , observes particle  $P_1$  at distance  $\frac{A}{\sqrt{2}}$  moving the right from mean position  $O$  and particle  $P_2$  at  $-\frac{\sqrt{3}}{2}A$  moving to the left from mean position  $O$ , as shown in figure. Find the value of  $\delta_2 - \delta_1$ .



- 22 Define S.H.M. What are its characteristics? At what distance from the mean position in S.H.M of amplitude  $r$ , the energy is half kinetic and half potential?
- 23 A body is executing a simple harmonic motion such that its potential energy is  $U_1$  at  $x$  and  $U_2$  at  $y$ . When the displacement is  $x + y$ , calculate the potential energy.
- 24 Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator.
- 25 A 10 kg collar is attached to a spring (spring constant 600 N/m), it slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 20 cm and released. What is the speed of the oscillation?
- 26 A particle is moving in a straight line with SHM of amplitude  $r$ . At a distance  $s$  from the mean position of motion, the particle receives a blow in the direction of motion which instantaneously doubles the velocity. Find the new amplitude.
- 27 If the ratio of the mechanical energy of a mass  $m$  whose oscillations are damped to that of its free oscillations is  $e^{-3t/m}$ , then determine the ratio of the amplitudes of the damped oscillations to that of free oscillations.
- 28 Draw (a) displacement-time graph of a particle executing SHM with phase angle  $\phi$  equal to zero (b) velocity-time graph and (c) acceleration-time graph of the particle.
29. Two particles are executing SHM with same amplitudes  $A$  and time period  $T$ . When one of the particles is located at extreme right, the other particle is located at mean position and moving towards left. From this instant, find the time after which both the particles will have same displacement from the equilibrium position.
30. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ( $t = 0$ ) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: ( $x$  is in cm and  $t$  is in s)
- (a)  $x = -2 \sin\left(3t + \frac{\pi}{3}\right)$  (b)  $x = \cos\left(\frac{\pi}{6} - t\right)$

- 31 Show the time period for vertical harmonic oscillations of the three-system shown in figure (a), (b) and (c) in the ratio of  $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$ . Spring constant of each spring of  $k$ .



32. A particle set to be in SHM having two types of energies potential and kinetic. The potential energy is on account of displacement of particle from mean position and kinetic energy is an account of velocity of the particle. At any instant of time  $t$ , these are –

$$P.E = U = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\sin^2 \omega t$$

$$K.E = K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2A^2\cos^2 \omega t$$

$$\text{and } T.E = P.E + K.E = \frac{1}{2}m\omega^2A^2 = \text{constant},$$

where,  $m$  = mass of particle.

(a) At what distance from the mean position (a fixed point) will K.E of particle would be twice of its P.E?

(b) What are the applications of this study in our day-to-day life?

33 A simple pendulum is hung in a stationary lift and its periodic time is  $T$ . What will be the effect on its periodic time  $T$  if

- the lift goes up with uniform velocity  $v$ ,
- the lift goes up with uniform acceleration  $a$ , and
- the lift comes down with uniform acceleration  $a$ ?

34. A horizontal spring block system of mass  $M$  executes simple harmonic motion. When the block is passing through its equilibrium position, an object of mass  $m$  is put on it and the two move together. Find the new amplitude and frequency of vibration.

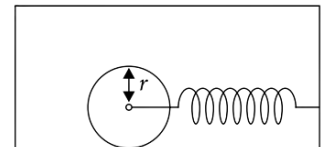
## 10. Long Answer Type Questions (LA)

35 (a) Draw a graph showing the variation of kinetic energy and potential energy of a particle executing SHM with its displacement from mean position.

(b) Show that total mechanical energy of a particle executing simple harmonic motion remains conserved with time, when dissipative forces are neglected.

36 A body oscillates with SHM along the  $X$ -axis. Its displacement varies with time according to the equation :  $x = (4.00 \text{ m})\cos(\pi t + \pi/4)$ . Calculate (a) displacement (b) velocity (c) acceleration at  $t = 1.00 \text{ s}$  (d) the maximum speed and maximum acceleration and (e) phase at  $t = 2.00 \text{ s}$ .

37. A solid cylinder of mass  $M$  is attached to a horizontal massless spring so that it can roll without slipping along a horizontal surface as shown in figure. The spring constant  $k$  is  $3 \text{ N m}^{-1}$ . If the system is released from rest at a point in which the spring is stretched by  $0.25 \text{ m}$ , find (a) the translational kinetic energy and (b) the rotational kinetic energy of the cylinder as it passes the equilibrium position.



(c) Show that under these conditions, the centre of mass of cylinder executes SHM with time period,  $T = 2\pi\sqrt{(3M/2k)}$ .

38. You are riding an automobile of mass  $3000 \text{ kg}$ . Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags  $15 \text{ cm}$  when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by  $50\%$  during one complete oscillation. Estimate the values of (a) the spring constant  $k$  and (b) the damping constant  $b$  for the spring and shock absorber system of one wheel, assuming that each wheel supports  $750 \text{ kg}$ .

## ANSWER

### MULTIPLE CHOICE QUESTIONS

01. (c) 02. (a) 03. (b) 04. (d) 05. (a) 06. (d) 07. (b) 08. (c) 09. (a) 10. (c) 11. (a) 12. (d)  
13. (a) 14. (a) 15. (d) 16. (b) 17. (b) 18. (a) 19. (b) 20. (a) 21. (c) 22. (b) 23. (d) 24. (a)  
25. (a) 26. (a) 27. (a) 28. (a) 29. (b) 30. (b) 31. (b) 32. (c) 33. (d) 34. (a) 35. (d) 36. (c)  
37. (d)

### CASE BASED STUDY:

38. (d) 39. (a) 40. (d) 41. (d) 42. (c) 43. (c) 44. (c) 45. (c) 46. (d)

### ASSERTION & REASONS

47. (c) 48. (b) 49. (b) 50. (b) 51. (a) 52. (a) 53. (c) 54. (a) 55. (c) 56. (b)